

© International Baccalaureate Organization 2022

All rights reserved. No part of this product may be reproduced in any form or by any electronic or mechanical means, including information storage and retrieval systems, without the prior written permission from the IB. Additionally, the license tied with this product prohibits use of any selected files or extracts from this product. Use by third parties, including but not limited to publishers, private teachers, tutoring or study services, preparatory schools, vendors operating curriculum mapping services or teacher resource digital platforms and app developers, whether fee-covered or not, is prohibited and is a criminal offense.

More information on how to request written permission in the form of a license can be obtained from <https://ibo.org/become-an-ib-school/ib-publishing/licensing/applying-for-a-license/>.

© Organisation du Baccalauréat International 2022

Tous droits réservés. Aucune partie de ce produit ne peut être reproduite sous quelque forme ni par quelque moyen que ce soit, électronique ou mécanique, y compris des systèmes de stockage et de récupération d'informations, sans l'autorisation écrite préalable de l'IB. De plus, la licence associée à ce produit interdit toute utilisation de tout fichier ou extrait sélectionné dans ce produit. L'utilisation par des tiers, y compris, sans toutefois s'y limiter, des éditeurs, des professeurs particuliers, des services de tutorat ou d'aide aux études, des établissements de préparation à l'enseignement supérieur, des fournisseurs de services de planification des programmes d'études, des gestionnaires de plateformes pédagogiques en ligne, et des développeurs d'applications, moyennant paiement ou non, est interdite et constitue une infraction pénale.

Pour plus d'informations sur la procédure à suivre pour obtenir une autorisation écrite sous la forme d'une licence, rendez-vous à l'adresse <https://ibo.org/become-an-ib-school/ib-publishing/licensing/applying-for-a-license/>.

© Organización del Bachillerato Internacional, 2022

Todos los derechos reservados. No se podrá reproducir ninguna parte de este producto de ninguna forma ni por ningún medio electrónico o mecánico, incluidos los sistemas de almacenamiento y recuperación de información, sin la previa autorización por escrito del IB. Además, la licencia vinculada a este producto prohíbe el uso de todo archivo o fragmento seleccionado de este producto. El uso por parte de terceros —lo que incluye, a título enunciativo, editoriales, profesores particulares, servicios de apoyo académico o ayuda para el estudio, colegios preparatorios, desarrolladores de aplicaciones y entidades que presten servicios de planificación curricular u ofrezcan recursos para docentes mediante plataformas digitales—, ya sea incluido en tasas o no, está prohibido y constituye un delito.

En este enlace encontrará más información sobre cómo solicitar una autorización por escrito en forma de licencia: <https://ibo.org/become-an-ib-school/ib-publishing/licensing/applying-for-a-license/>.

Mathematics: analysis and approaches
Higher level
Paper 2

Monday 9 May 2022 (morning)

Candidate session number

--	--	--	--	--	--	--	--	--	--

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.



Please **do not** write on this page.

Answers written on this page
will not be marked.



16EP02

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

The number of hours spent exercising each week by a group of students is shown in the following table.

Exercising time (in hours)	Number of students
2	5
3	1
4	4
5	3
6	x

The median is 4.5 hours.

- (a) Find the value of x . [2]
- (b) Find the standard deviation. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

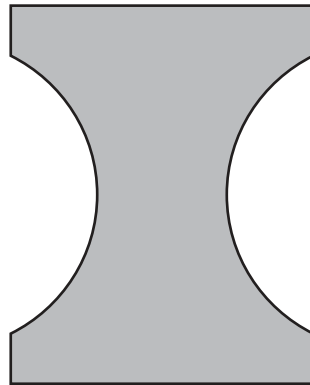


Turn over

2. [Maximum mark: 6]

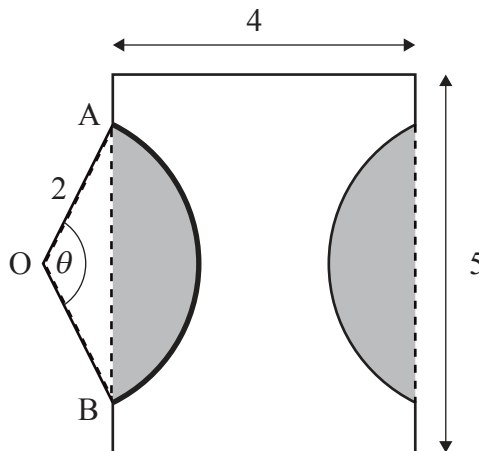
A company is designing a new logo. The logo is created by removing two equal segments from a rectangle, as shown in the following diagram.

diagram not to scale



The rectangle measures 5 cm by 4 cm. The points A and B lie on a circle, with centre O and radius 2 cm, such that $\widehat{AOB} = \theta$, where $0 < \theta < \pi$. This information is shown in the following diagram.

diagram not to scale



(a) Find the area of one of the shaded segments in terms of θ . [3]

(b) Given that the area of the logo is 13.4 cm^2 , find the value of θ . [3]

(This question continues on the following page)



(Question 2 continued)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



16EP05

Turn over

3. [Maximum mark: 6]

A discrete random variable, X , has the following probability distribution:

x	0	1	2	3
$P(X=x)$	0.41	$k - 0.28$	0.46	$0.29 - 2k^2$

- (a) Show that $2k^2 - k + 0.12 = 0$. [1]
- (b) Find the value of k , giving a reason for your answer. [3]
- (c) Hence, find $E(X)$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



4. [Maximum mark: 7]

A particle moves along a straight line so that its velocity, $v \text{ m s}^{-1}$, after t seconds is given by $v(t) = e^{\sin t} + 4 \sin t$ for $0 \leq t \leq 6$.

- (a) Find the value of t when the particle is at rest. [2]
- (b) Find the acceleration of the particle when it changes direction. [3]
- (c) Find the total distance travelled by the particle. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



16EP07

Turn over

5. [Maximum mark: 6]

Let A and B be two independent events such that $P(A \cap B') = 0.16$ and $P(A' \cap B) = 0.36$.

(a) Given that $P(A \cap B) = x$, find the value of x . [4]

(b) Find $P(A' | B')$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



16EP08

6. [Maximum mark: 6]

Consider the function $f(x) = 2^x - \frac{1}{2^x}$, $x \in \mathbb{R}$.

(a) Show that f is an odd function. [2]

The function g is given by $g(x) = \frac{x-1}{x^2-2x-3}$, where $x \in \mathbb{R}$, $x \neq -1$, $x \neq 3$.

(b) Solve the inequality $f(x) \geq g(x)$. [4]

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....



16EP09

Turn over

7. [Maximum mark: 9]

Consider the vectors \mathbf{a} and \mathbf{b} such that $\mathbf{a} = \begin{pmatrix} 12 \\ -5 \end{pmatrix}$ and $|\mathbf{b}| = 15$.

(a) Find the possible range of values for $|\mathbf{a} + \mathbf{b}|$. [2]

Consider the vector \mathbf{p} such that $\mathbf{p} = \mathbf{a} + \mathbf{b}$.

(b) Given that $|\mathbf{a} + \mathbf{b}|$ is a minimum, find \mathbf{p} . [2]

Consider the vector \mathbf{q} such that $\mathbf{q} = \begin{pmatrix} x \\ y \end{pmatrix}$, where $x, y \in \mathbb{R}^+$.

(c) Find \mathbf{q} such that $|\mathbf{q}| = |\mathbf{b}|$ and \mathbf{q} is perpendicular to \mathbf{a} . [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



8. [Maximum mark: 4]

Consider the equation $kx^2 - (k + 3)x + 2k + 9 = 0$, where $k \in \mathbb{R}$.

- (a) Write down an expression for the product of the roots, in terms of k . [1]
- (b) Hence or otherwise, determine the values of k such that the equation has one positive and one negative real root. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



16EP11

Turn over

9. [Maximum mark: 7]

Mary, three female friends, and her brother, Peter, attend the theatre. In the theatre there is a row of 10 empty seats. For the first half of the show, they decide to sit next to each other in this row.

(a) Find the number of ways these five people can be seated in this row. [3]

For the second half of the show, they return to the same row of 10 empty seats. The four girls decide to sit at least one seat apart from Peter. The four girls do not have to sit next to each other.

(b) Find the number of ways these five people can now be seated in this row. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 20]

Consider the function $f(x) = \sqrt{x^2 - 1}$, where $1 \leq x \leq 2$.

- (a) Sketch the curve $y = f(x)$, clearly indicating the coordinates of the endpoints. [2]
- (b) (i) Show that the inverse function of f is given by $f^{-1}(x) = \sqrt{x^2 + 1}$.
- (ii) State the domain and range of f^{-1} . [5]

The curve $y = f(x)$ is rotated 2π about the y -axis to form a solid of revolution that is used to model a water container.

- (c) (i) Show that the volume, $V \text{ m}^3$, of water in the container when it is filled to a height of h metres is given by $V = \pi \left(\frac{1}{3}h^3 + h \right)$.
- (ii) Hence, determine the maximum volume of the container. [5]

At $t = 0$, the container is empty. Water is then added to the container at a constant rate of $0.4 \text{ m}^3 \text{ s}^{-1}$.

- (d) Find the time it takes to fill the container to its maximum volume. [2]
- (e) Find the rate of change of the height of the water when the container is filled to half its maximum volume. [6]



Do **not** write solutions on this page.

11. [Maximum mark: 16]

A bakery makes two types of muffins: chocolate muffins and banana muffins.

The weights, C grams, of the chocolate muffins are normally distributed with a mean of 62 g and standard deviation of 2.9 g.

- (a) Find the probability that a randomly selected chocolate muffin weighs less than 61 g. [2]
- (b) In a random selection of 12 chocolate muffins, find the probability that exactly 5 weigh less than 61 g. [2]

The weights, B grams, of the banana muffins are normally distributed with a mean of 68 g and standard deviation of 3.4 g.

Each day 60% of the muffins made are chocolate.

On a particular day, a muffin is randomly selected from all those made at the bakery.

- (c) (i) Find the probability that the randomly selected muffin weighs less than 61 g.
- (ii) Given that a randomly selected muffin weighs less than 61 g, find the probability that it is chocolate. [7]

The machine that makes the chocolate muffins is adjusted so that the mean weight of the chocolate muffins remains the same but their standard deviation changes to σ g. The machine that makes the banana muffins is not adjusted. The probability that the weight of a randomly selected muffin from these machines is less than 61 g is now 0.157.

- (d) Find the value of σ . [5]



Do **not** write solutions on this page

12. [Maximum mark: 19]

Consider the differential equation $x^2 \frac{dy}{dx} = y^2 - 2x^2$ for $x > 0$ and $y > 2x$. It is given that $y = 3$ when $x = 1$.

(a) Use Euler’s method, with a step length of 0.1, to find an approximate value of y when $x = 1.5$. [4]

(b) Use the substitution $y = vx$ to show that $x \frac{dv}{dx} = v^2 - v - 2$. [3]

(c) (i) By solving the differential equation, show that $y = \frac{8x + x^4}{4 - x^3}$.

(ii) Find the actual value of y when $x = 1.5$.

(iii) Using the graph of $y = \frac{8x + x^4}{4 - x^3}$, suggest a reason why the approximation given by Euler’s method in part (a) is not a good estimate to the actual value of y at $x = 1.5$. [12]

References:

© International Baccalaureate Organization 2022



16EP15

Please **do not** write on this page.

Answers written on this page
will not be marked.



16EP16